

where  $\beta$  is the frequency of non-linear vibrations, which is the same for the vertical vibrations and the bending and twisting vibrations.

The results of the analysis of the vibrations subject to frequency modulation only based on the data in Tables 1 and 2 are collected in Table 3. It can be seen that the domain of existence of the solution determined by (4.2) and (4.3) is reduced to a point for every skew-symmetric characteristic form of vibrations (such vibrations are possible for one value of the frequency  $\kappa$ ), and either extends to infinity or disappears completely for every symmetric form of characteristic vibrations.

Therefore, in the case of similar frequencies the non-linear vibrations (4.3) subject to frequency modulation can be realized for the given values  $\omega_{0m}$ ,  $\Omega_{0m}$  and  $\sigma$  only, while, as can be seen from (4.1), in the case of internal resonance such vibrations can be excited for any values of  $\omega_{0m}$ ,  $\Omega_{0m}$  and  $\sigma$ . These results are consistent with the graphs in Figs.2-4.

Finally, we mention that the regime of vibrations in the case of similar frequencies turns out to be more stable under variations of the level of mistuning than the regime of vibrations in the case of internal resonance. This is evident from the graphs of the time dependence of the amplitude function envelopes for the following three values of detuning:  $\varepsilon^2\sigma = -0.5 \times 10^{-4}$  (the broken line),  $\sigma = 0$  (the solid line), and  $\varepsilon^2\sigma = 0.5 \times 10^{-4}$  (the dash-dot line), which are given in Fig.6 for the case of internal resonance ( $a_1(T_1) = 0.02$ ) and the case of similar frequencies ( $a_1(T_2) \geq 0.02$ ). One can see that even a slight violation of the resonance condition  $\omega_0 = 2\Omega_0$  results in the maxima of the amplitude functions being immediately reduced to zero.

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## STRESSES IN ELASTIC CONICAL TUBES OF TRANSVERSELY ISOTROPIC MATERIALS WITH SPHERICAL ANISOTROPIES UNDER TEMPERATURE AND FORCE LOADINGS\*

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Analytic solutions are proposed for a number of new problems on determining the state of stress of a transversely-isotropic hollow cone with spherical anisotropy. An exact solution of the problem of the axisymmetric deformation of a long conical tube (or continuous cone) from an elastic transversely-isotropic material with spherical anisotropy subjected to an axial force is obtained in a spherical coordinate system  $R, \varphi, \theta$ ; the material axis of symmetry is directed along the spherical radius  $R$ . A rigorous solution is given of the problem of the uniform heating of a conical tube of transversely-isotropic material with spherical anisotropy for particular values of Poisson's ratios; the material axis of symmetry is directed along the  $\theta$ -axis. For arbitrary Poisson's ratios an asymptotic solution is found for the temperature problem for a tube with small conicity.

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The general approach for determining the state of stress of an orthotropic hollow cone with a spherical anisotropy /1/ uses the method of separation of variables, which enables the problem to be reduced to a one-dimensional one, with subsequent solution by the method of discrete orthogonalization. Analytic solutions of problems on the deformation of anisotropic cones are presented in the literature for the case when the material has cylindrical anisotropy. In particular, exact solutions are constructed for the problems of the stretching a cone /2/ and of the selfsimilar heating of a hollow cone /3/ when the axes of physical and geometric symmetry of the elastic transversely-isotropic material are collinear.

1. *A conical tube subjected to an axial force.* The elastic equilibrium of an anisotropic tube bounded by coaxial circular conical surfaces with a single apex is considered. We take the apex as the origin of the spherical system  $R, \varphi, \theta$  /4/. The inner surface is described by the equation  $\theta = \theta_1$ , and the outer one by the equation  $\theta = \theta_2$ . Generally speaking the tube has ends  $R = R_0 > 0$  and  $R = R_1 > R_0$ .

If the arc length  $R_1(\theta_2 - \theta_1)$  is small compared with the segment  $R_1 - R_0$  of the tube generatrix, then we call such a tube long. In this case edge effects can be ignored and the boundary conditions at the ends  $R = R_0$  and  $R = R_1$  can be specified following the Saint-Venant principle.

Henceforth we will ascribe the subscripts 1, 2, 3, respectively, to the directions  $R, \varphi, \theta$ .

The connection between the stresses  $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{13}$  and strains  $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{13}$  in the transversely-isotropic material considered with spherical anisotropy can be represented as follows /2/

$$\begin{aligned} E_1 \varepsilon_{11} &= k \sigma_{11} - kv'(\sigma_{22} + \sigma_{33}), & E_2 \varepsilon_{22} &= \sigma_{22} - v \sigma_{33} - kv' \sigma_{11} \\ E_2 \varepsilon_{33} &= \sigma_{33} - v \sigma_{22} - kv' \sigma_{11}, & E_2 \varepsilon_{13} &= \gamma \sigma_{13} \\ \gamma &= E_2/G, & k &= E_2/E_1 \end{aligned} \quad (1.1)$$

The axis 1 is the material axis of symmetry,  $E_2$  and  $E_1$  are the elastic moduli in the directions along the axes 2 and 1, respectively,  $G$  is the shear modulus, and  $v$  and  $v'$  are Poisson's ratios.

The solution of the problem will be sought in the form

$$\sigma_{ij} = \frac{\tau_{ij}(\theta)}{R^2}, \quad \varepsilon_{ij} = \frac{e_{ij}(\theta)}{R^2}, \quad u_1 = \frac{H_1(\theta)}{R}, \quad u_3 = \frac{H_3(\theta)}{R} \quad (1.2)$$

where  $u_1, u_3$  are the displacements along axes 1 and 3.

The Cauchy dependences reduce to the relations

$$e_{11} = -H_1', \quad e_{22} = H_1 + \text{ctg } \theta H_3, \quad e_{33} = H_3' + H_1, \quad e_{13} = H_1' - 2H_3$$

The prime denotes differentiation with respect to the argument  $\theta$ .

Taking (1.2) into account, we can write the equilibrium equations for the case of axisymmetric strain in the spherical coordinate system /4/

$$\begin{aligned} \tau_{13}' - \tau_{22} - \tau_{33} + \tau_{13} \text{ctg } \theta &= 0 \\ \tau_{33}' + \tau_{13} + (\tau_{33} - \tau_{22}) \text{ctg } \theta &= 0 \end{aligned} \quad (1.3)$$

and the strain compatibility equations taking the Cauchy relationships into account

$$\left( \frac{e_{22} + e_{11}}{\text{ctg } \theta} \right)' = e_{33} + e_{11}, \quad e_{13} = -e_{11}' - 2 \frac{e_{11} + e_{22}}{\text{ctg } \theta} \quad (1.4)$$

The boundary conditions on the side surfaces of the tube are written as follows:

$$\tau_{13}(\theta_1) = \tau_{13}(\theta_2) = 0, \quad \tau_{33}(\theta_1) = \tau_{33}(\theta_2) = 0 \quad (1.5)$$

The equilibrium condition /4/ ( $Q$  is the magnitude of the axial force)

$$2\pi \int_{\theta_1}^{\theta_2} (\tau_{11} \cos \theta - \tau_{13} \sin \theta) \sin \theta \, d\theta = Q \quad (1.6)$$

should be satisfied on the endfaces  $R = R_0$  and  $R = R_1$ .

Eqs.(1.3) are satisfied automatically for

$$\tau_{33} = \text{ctg } \theta \tau_{13}, \quad \tau_{22} = \tau_{13}' \quad (1.7)$$

Conditions (1.5) are obviously identical.

Taking account of relationships (1.1) and (1.7), we write the general integral of the first equation in (1.4) in the form ( $B$  is an undetermined constant)

$$(k - kv') \tau_{11} = -(1 - kv') (\tau_{22} + \tau_{33}) + (k - kv') B \cos \theta \tag{1.8}$$

The function  $\tau_{13}$  is determined from the second relationship in (1.4), which reduces to the equation

$$\frac{1}{\sin \theta} (\sin \theta \tau_{13})' - \left( \omega_1 + \frac{1}{\sin^2 \theta} \right) \tau_{13} = B \omega_2 \sin \theta \tag{1.9}$$

$$\omega_1 = \frac{\gamma - 2(1 + \nu)}{\mu}, \quad \omega_2 = \frac{k - 2kv'}{\mu}, \quad \mu = \frac{k - (kv')^2}{k - kv'}$$

when (1.7) and (1.8) are taken into account.

The case  $k > 1$  is of interest. For  $k > 1$  it can be assumed that  $4\omega_1 > 1$  for a material with a quite definite anisotropy.

We will find the solution of (1.9). A solution of the appropriate homogeneous differential equation is an associated Legendre spherical function of the first kind  $P_{1/2, 1/2}^1(\cos \theta)$ ,  $s = 1/2 \sqrt{4\omega_1 - 1}$  (a function of the cone)  $/5/$ . It has no zeros in the interval  $|\cos \theta| < 1$ . Therefore, the solution of (1.9) is the following  $/6/$ :

$$\tau_{13} = B \omega_2 Z(\theta),$$

$$Z(\theta) = P_{1/2, 1/2}^1(\cos \theta) \left\{ C_1 - C_2 \int_0^\theta U(\alpha) d\alpha + \int_0^\theta U(\alpha) \int_{\theta_1}^\alpha V(\beta) d\beta d\alpha \right\}$$

$$U(\alpha) = \{ \sin \alpha [P_{1/2, 1/2}^1(\cos \alpha)]^2 \}^{-1}, \quad V(\beta) = \sin^2 \beta P_{1/2, 1/2}^1(\cos \beta)$$

The constants  $C_1$  and  $C_2$  are determined from the first two boundary conditions in (1.5)

$$C_1 = 0, \quad C_2 = - \int_{\theta_1}^{\theta_2} U(\alpha) \int_{\theta_1}^\alpha V(\beta) d\beta d\alpha \left[ \int_{\theta_1}^{\theta_2} U(\alpha) d\alpha \right]^{-1}$$

The constant  $B$  is determined from condition (1.6) which reduces, when the conditions  $\tau_{13}(\theta_1) = \tau_{13}(\theta_2) = 0$  are taken into account, to the equality

$$B \left[ \frac{1}{3} (\cos^3 \theta_1 - \cos^3 \theta_2) - \frac{k + 1 - 2kv'}{k - kv'} \omega_2 \int_{\theta_1}^{\theta_2} Z(\theta) \sin^2 \theta d\theta \right] = \frac{Q}{2\pi} \tag{1.10}$$

We will write the solution of (1.9) as it applies to a continuous cone with the side surface  $\theta = \theta_2$ :

$$Z(\theta) = P_{1/2, 1/2}^1(\cos \theta) \left( C_1 + \int_0^\theta U(\alpha) \int_0^\alpha V(\beta) d\beta d\alpha \right), \quad \tau_{13} = B \omega_2 Z(\theta)$$

The constant  $C_1$  is determined from the condition  $\tau_{13}(\theta_2) = 0$ :

$$C_1 = - \int_0^{\theta_2} U(\alpha) \int_0^\alpha V(\beta) d\beta d\alpha$$

Note that  $Z(0) = 0$  since  $P_{1/2, 1/2}^1(1) = 0$  and the function

$$U(\theta) \int_0^\theta V(\beta) d\beta$$

has no singularity at  $\theta = 0$ .

The constant  $B$  is determined from (1.10) in which we should set  $\theta_1 = 0$ .

**2. Uniform heating of a conical tube.** Let an elastic transversely-isotropic conical tube be subjected to uniform heating or cooling. The body possesses spherical anisotropy, where the material axis of symmetry is directed along the  $\theta$ -axis.

In this case the connection between the stress and strain is described by the relationships

$$\begin{aligned}
 E_2 \varepsilon_{11} &= \sigma_{11} - \nu \sigma_{22} - k \nu' \sigma_{33} + E_2 \alpha_2 \Delta T & (2.1) \\
 E_2 \varepsilon_{22} &= \sigma_{22} - \nu \sigma_{11} - k \nu' \sigma_{33} + E_2 \alpha_2 \Delta T \\
 E_2 \varepsilon_{33} &= k \sigma_{33} - k \nu' (\sigma_{11} + \sigma_{22}) + E_2 \alpha_3 \Delta T \\
 E_2 \varepsilon_{13} &= \gamma \sigma_{13}, \quad k = E_2' / E_3, \quad \gamma = E_2 / G
 \end{aligned}$$

where  $E_3$  is the elastic modulus along the axis  $3$  ( $\theta$ ),  $\alpha_2, \alpha_3$  are coefficients of linear expansion along the axes  $2$  and  $3$ , and  $\Delta T$  is the body temperature ( $\Delta T = \text{const}$ ).

The solution of the problem is sought in selfsimilar form

$$\sigma_{ij} = \sigma_{ij}(y), \quad y = \text{ctg } \theta \tag{2.2}$$

We write the equilibrium equation in a spherical system of coordinates taking (2.2) /4/ and the strain compatibility equation into account

$$-(1 + y^2) \sigma_{13}' + 2\sigma_{11} - \sigma_{22} - \sigma_{33} + y \sigma_{13} = 0 \tag{2.3}$$

$$\begin{aligned}
 -(1 + y^2) \sigma_{33}' + y (\sigma_{33} - \sigma_{22}) &= -3\sigma_{13} \\
 (1 + y^2) \varepsilon_{11}' = -\varepsilon_{13}, \quad \varepsilon_{33} - \varepsilon_{11} &= -(1 + y^2) [(e_{22} - \varepsilon_{11})/y]' & (2.4)
 \end{aligned}$$

where the prime denotes differentiation with respect to the argument  $y$ .

On the tube side surfaces  $y = a_1 = \text{ctg } \theta_1$  and  $y = a_2 = \text{ctg } \theta_2$  we have

$$\sigma_{33}(a_1) = \sigma_{33}(a_2) = 0, \quad \sigma_{13}(a_1) = \sigma_{13}(a_2) = 0 \tag{2.5}$$

The condition that the equivalent force in the sections  $R = \text{const}$  equal zero is satisfied automatically when the equilibrium equations and boundary conditions (2.5) are satisfied.

We will first examine the special case when  $k \nu' = \nu = 1/2$ .

Setting  $\sigma_{13} \equiv 0$ , we have from the first equation in (2.3)

$$2\sigma_{11} = \sigma_{22} + \sigma_{33} \quad \text{or} \quad \varepsilon_{11} = \alpha_2 \Delta T \tag{2.6}$$

The first equation in (2.4) is satisfied automatically.

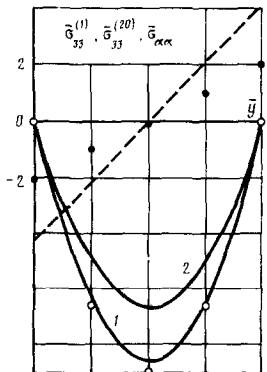
Substituting the second relationship from (2.3) and (2.6) into the second equation of (2.4) we obtain

$$\begin{aligned}
 \sigma_{33}'' - \frac{2 + y^2}{y(1 + y^2)} \sigma_{33}' - \frac{4}{3}(k - 1) \frac{y^2}{(1 + y^2)^2} \sigma_{33} &= \frac{4}{3} \frac{y^2}{(1 + y^2)^2} P \\
 P &= E_2 (\alpha_3 - \alpha_2) \Delta T
 \end{aligned} \tag{2.7}$$

Taking the condition  $\sigma_{33}(a_1) = 0$  into account we represent (2.7) in the form

$$\begin{aligned}
 \sigma_{33} &= D [Q(y) - Q(a_1)] + \frac{4}{3} P [W(y) - W(a_1)] + \\
 &\quad - \frac{4}{3} (k - 1) \int_{a_1}^y \xi^2 (1 + \xi^2)^{-1/2} \int_{a_1}^{\xi} (1 + t^2)^{-3/2} \sigma_{33} dt d\xi \\
 2Q(y) &= y \sqrt{1 + y^2} - \ln(y + \sqrt{1 + y^2}), \\
 2W(y) &= y^2 - \ln(1 + y^2)
 \end{aligned} \tag{2.8}$$

The constant  $D$  is determined from the condition  $\sigma_{33}(a_2) = 0$ .



Eq.(2.8) is solved by an iteration method that is used when solving equations of the Volterra type

$$\begin{aligned}
 F(y) &= \int_{a_1}^y \xi^2 (1 + \xi^2)^{-1/2} \int_{a_1}^{\xi} (1 + t^2)^{-3/2} \sigma_{33}^{(n-1)} dt d\xi \\
 \sigma_{33}^{(0)}(y) &\equiv 0, \quad \sigma_{33}^{(n)}(y) = \\
 &D [Q(y) - Q(a_1)] + \Phi(y) \\
 \Phi(y) &= 4/3 P [W(y) - \\
 &W(a_1)] + 4/3 (k - 1) F(y), \\
 D &= -\Phi(a_2) [Q(a_2) - Q(a_1)]^{-1}
 \end{aligned}$$

The superscript  $n$  denotes the number of the approximate solution. The stress  $\sigma_{22}$  is determined from the second formula in (2.3) in which we should set

$$\sigma_{33}' = y^2 (1 + y^2)^{-1/2} \left[ D + \frac{4}{3} P y (1 + y^2)^{-1/2} + \frac{4}{3} (k-1) \int_{a_1}^y (1 + \xi^2)^{-1/2} \sigma_{33} d\xi \right]$$

The function  $\sigma_{11}$  is determined from relationship (2.6).

The iteration method mentioned is realized on an electronic computer. Numerical values are obtained for twenty approximate solutions of  $\sigma_{33}$ . The results show that  $\sigma_{33}^{(1)}$  is practically identical with  $\sigma_{33}^{(20)}$  for thin-walled tubes ( $(a_1 - a_2)/a_2 \ll 1$ ). Graphs of the functions

$\bar{\sigma}_{33}^{(20)} = 5 \times 10^2 \sigma_{33}^{(20)}(y)/P$  (the solid line 1) and  $\bar{\sigma}_{33}^{(1)} = 5 \times 10^2 \sigma_{33}^{(1)}(y)/P$  (open circles) are presented in the figure for a thick tube for  $a_1 = 8.4$ ,  $a_2 = 6$ , and  $k = 4$ , where  $\bar{y} = (y - a_1)/(a_2 - a_1)$ .

The distribution of the stress  $\bar{\sigma}_{33} = 10^4 \sigma_{33}(y)/P$  (solid line 2),  $\bar{\sigma}_{22} = 10^2 \sigma_{22}(y)/P$  (the dashed line) and  $\bar{\sigma}_{11} = 10^2 \sigma_{11}(y)/P$  (the dark points) is also shown for  $a_1 = 6.402$ ,  $a_2 = 6$  and  $k = 4$ .

We will investigate the case when Poisson's ratios are arbitrary and the tube possesses small conicity. It is assumed for such a tube that  $a_2^{-2} \ll 1$  and the shear stress  $\sigma_{13}$  is small compared with the other stresses. Consequently, it is permissible to discard the terms  $\sigma_{13}$  and  $\varepsilon_{13}$  respectively, in the second equation in (2.3) and the first equation in (2.4).

Moreover, we set  $1 + y^2 \approx y^2$  in (2.3) and (2.4). We have

$$\begin{aligned} -y^2 \sigma_{13}' + 2\sigma_{11} - \sigma_{22} - \sigma_{33} + y\sigma_{13} &= 0 \\ -y^2 \sigma_{33}' + y(\sigma_{33} - \sigma_{22}) &= 0 \\ \varepsilon_{11}' = 0, \quad \varepsilon_{33} - \varepsilon_{11} &= -y^2 [(\varepsilon_{22} - \varepsilon_{11})/y]' \end{aligned} \quad (2.9)$$

The solution of system (2.1) and (2.9) under the conditions (2.5) will be called asymptotic.

We will write the third equation in (2.9) in the form ( $A$  is a constant)

$$E_2 \varepsilon_{11} = E_2 \alpha_2 \Delta T + A \quad \text{or} \quad \sigma_{11} - \nu \sigma_{22} - kv' \sigma_{33} = A \quad (2.10)$$

Integrating the second and fourth equations in (2.9) taking (2.10) and the first two boundary conditions in (2.5) into account, we obtain

$$\begin{aligned} \sigma_{33} &= \Pi (Cy^{\omega+1} + Dy^{1-\omega} - 1), \quad \sigma_{22} = \Pi (-\omega Cy^{\omega+1} + \omega Dy^{1-\omega} - 1) \\ \Pi &= \frac{P \pm (\nu - kv') A}{(1 - \nu^2)(\omega^2 - 1)}, \quad \omega = \left[ \frac{k - (kv')^2}{1 - \nu^2} \right]^{1/2} \\ D &= \frac{a_2^{\omega+1} - a_1^{\omega+1}}{a_1 a_2} \left[ \left( \frac{a_2}{a_1} \right)^\omega - \left( \frac{a_1}{a_2} \right)^\omega \right]^{-1}, \quad C = \frac{1 - Da_1^{1-\omega}}{a_1^{\omega-1}} \end{aligned} \quad (2.11)$$

It was assumed that  $\omega \neq 1$  when deriving (2.11).

The function  $\sigma_{13}$  is determined from the first equation in (2.9)

$$\begin{aligned} \sigma_{13} &= By - \frac{A}{y} - \Pi \frac{f(y)}{y}, \quad f(y) = \frac{\beta_- C}{\omega - 1} y^{\omega+1} - \frac{\beta_+ D}{\omega + 1} y^{1-\omega} \pm (1 - \nu - kv') \\ \beta_{\pm} &= 1 - 2kv' \pm \omega(1 - 2\nu) \end{aligned}$$

The constants  $B$  and  $A$  are determined from the last two conditions in (2.5), we have, in particular, for  $\nu = kv'$

$$\sigma_{13} = (1 - 2\nu) \sigma_{33}/y$$

This last formula shows that the right-hand sides of the second equation in (2.3) and the first equation in (2.4) can actually be neglected if  $a_2^{-2} \ll 1$ .

The solutions obtained in this paper for the problem of temperature heating of a hollow cone show that the state of stress of a conical shell of linearly variable thickness ( $(\theta_2 - \theta_1)/\theta_1 \ll 1$ ) depends only slightly on the ratio  $k = E_2/E_3$ .

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## A HOLLOW ELLIPSOIDAL NEEDLE IN AN ORTHOTROPIC ELASTIC MEDIUM\*

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The problem of the stress distribution on the surface of a hollow ellipsoidal needle in an orthotropic elastic medium and a homogeneous external field is solved. Explicit expressions are obtained for the stresses on the needle surface in terms of the elastic constants of the medium and parameters of the ellipsoid in a local system of coordinates connected to the normal to the surface at each point of the needle. The general solution of the problem of the stress concentration on an ellipsoidal inhomogeneity /1/ and the passage to the limit cases of an ellipsoidal cavity based on the presence of small parameters /2/ is used.

1. Consider a hollow ellipsoidal needle, i.e., an ellipsoidal cavity, one of whose dimensions is large compared with the other two, in an orthotropic unbounded elastic medium subjected to an external uniform field  $\sigma_0^{\alpha\beta}$ . The equation of the ellipsoid is written in

the form

$$x_1^2 a_1^{-2} + x_2^2 a_2^{-2} + x_3^2 a_3^{-2} = 1, \quad a_1 \gg a_2 > a_3 \quad (1.1)$$

in an  $(x_1, x_2, x_3)$  system of coordinates rigidly connected to the ellipsoid.

We will assume that the axes of elastic symmetry of the external orthotropic medium coincide with the axes of the ellipsoid. Then the tensor of the elastic constants of the medium  $c^{\alpha\beta\lambda\mu}$  has nine non-zero components that are denoted according to the usual rule /3/ by

$$\begin{aligned} c^{\alpha\alpha\beta\beta} &= c_{\alpha\beta} \quad (\alpha, \beta = 1, 2, 3) \\ c^{2323} &= c_{44}, \quad c^{3131} = c_{55}, \quad c^{1212} = c_{66} \end{aligned} \quad (1.2)$$

The stresses  $\sigma^{\alpha\beta}(\mathbf{n})$  on the surface of an ellipsoidal cavity in a uniform external field  $\sigma_0^{\lambda\mu}$  have the form

$$\sigma^{\alpha\beta}(\mathbf{n}) = F_{\lambda\alpha}^{\alpha\beta}(\mathbf{n}) \sigma_0^{\lambda\mu}, \quad F_{\lambda\mu}^{\alpha\beta}(\mathbf{n}) = B^{\alpha\beta\lambda\mu}(\mathbf{n}) B_{\lambda\mu}^{-1} \quad (1.3)$$

where  $\mathbf{n} = (n_1, n_2, n_3)$  is the normal to the ellipsoid surface.

The tensor stress concentration coefficient  $F_{\lambda\mu}^{\alpha\beta}(\mathbf{n})$  can be represented in the form of the product of two factors. The first of them, the tensor  $B^{\alpha\beta\lambda\mu}(\mathbf{n})$  depends only on the elastic constants of the medium and the inclusion and on the normal  $\mathbf{n}$  to the inclusion surface, and remains finite for any passages to the limit. For a cavity the tensor  $B(\mathbf{n})$  has the form /1/

$$B^{\alpha\beta\lambda\mu}(\mathbf{n}) = c^{\alpha\beta\lambda\mu} - c^{\alpha\beta\lambda\rho} K_{\lambda\rho\eta\nu}(\mathbf{n}) c^{\eta\nu\lambda\mu} \quad (1.4)$$

where the tensor  $K(\mathbf{n})$  for an orthotropic medium is constructed explicitly in terms of the Fourier transform of Green's tensor of a homogeneous medium. The expressions for the components of  $K(\mathbf{n})$  in terms of the elastic constants of an orthotropic medium and the coordinates

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